(*a*) The longitudinal stress throughout the tube is zero.

(*b*) The cylinder is considered infinitely long, since the center portion is far enough from the ends to be free of any end effect. The longitudinal strain, therefore, must be constant with respect to r.

(*c*) The pressure required to produce full plastic flow in the tube is given by Eq. (4), i.e., $P_0 = 1.08\,\sigma_y \log W$ where W is the initial diameter ratio.

(*d*) If a cylinder is in the fully plastic condition (subject to an internal pressure P_0) and is then subjected to an external pressure, P_{ex}, in order to maintain the fully plastic condition and equilibrium, the internal pressure, P_{in}, must be increased by an amount equal to P_{ex}.

$$P_{in} = P_0 + P_{ex} \tag{7}$$

1. *Elastic region.* The elastic portion of a partially plastic cylinder is considered as a separate cylinder of internal radius ρ (elastic-plastic interface radius) and external radius b. The radial stress and therefore the internal pressure at ρ is equal to that required to produce elastic breakdown at ρ. Therefore, from Eq. (1):

$$\sigma_{r_\rho} = -\sigma_y \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}} \tag{8}$$

The stresses throughout the elastic region are then obtained from Eq. (8) and the Lamé equations

$$\sigma_{re} = \frac{\sigma_y \rho^2}{\sqrt{(3b^4 + \rho^4)}} \left(1 - \frac{b^2}{r^2}\right) \tag{9}$$

$$\sigma_{te} = \frac{\sigma_y \rho^2}{\sqrt{(3b^4 + \rho^4)}} \left(1 + \frac{b^2}{r^2}\right) \tag{10}$$

where $\rho \leq r \leq b$.

Strains in the elastic region are obtained from the above and the generalized Hookes' law.

$$\epsilon_{re} = \frac{\sigma_y \rho^2}{r^2 E \sqrt{(3b^4 + \rho^4)}} [(1 - \mu) r^2 - (1 + \mu) b^2] \tag{11}$$

$$\epsilon_{te} = \frac{\sigma_y \rho^2}{r^2 E \sqrt{(3b^4 + \rho^4)}} [(1 - \mu) r^2 + (1 + \mu) b^2] \tag{12}$$

$$\epsilon_z = -\frac{2\mu\, \sigma_y\, \rho^2}{E \sqrt{(3b^4 + \rho^4)}} \tag{13}$$

2. *Plastic region.* The plastic portion of the cylinder, i.e. where $a \leq r \leq \rho$, may be considered as a separate, fully plastic cylinder acted on by an internal pressure equal to $-\sigma_{r\rho}$ at some radius, r, and an external pressure $-\sigma_{r_\rho}$ at ρ. Therefore, from Eq. (7)

$$\sigma_{rp} = \sigma_{r_{\rho'}} + \sigma_{r_\rho} \tag{14}$$

where $\sigma_{r_{\rho'}}$ equals $-P_{0'}$ which is the pressure required to produce full plastic flow in section considered.

From Eq. (4)

$$\sigma_{r\rho'} = 1.08\,\sigma_y \ln \frac{\rho}{r} \tag{15}$$

Substituting σ_{r_ρ} and $\sigma_{r_{\rho'}}$ from Eqs. (8) and (15), respectively, into Eq. (14) yields for the radial stress at radius r in the plastic region.

$$\sigma_{rp} = -\sigma_y \left[1.08 \ln \frac{\rho}{r} + \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}}\right] \tag{16}$$

The well known equilibrium equation for a thick-wall cylinder is

$$\sigma_t = \sigma_r + r \frac{\mathrm{d}\sigma r}{\mathrm{d}r} \tag{17}$$

From Eqs. (16) and (17), the tangential stress in the plastic region becomes:

$$\sigma_{tp} = \sigma_y \left[-1.08 \ln \frac{\rho}{r} + 1.08 - \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}}\right] \tag{18}$$

The use of the empirical coefficient 1.08 in Eqs. (16) and (18) leads to a very slight discontinuity in the σ_t distribution at the elastic–plastic interface. This is due to the coefficient varying somewhat with the elastic–plastic interface location. However, since the error is small, for the sake of simplicity it can be assumed that the coefficient is constant and independent of ρ.

To determine the strains in the plastic region, it is assumed that the only change in volume in the plastic region is elastic in nature and is given by:

$$\epsilon_r + \epsilon_t + \epsilon_z = \frac{1 - 2\mu}{E} (\sigma_r + \sigma_t + \sigma_z) \tag{19}$$

Defining ϵ_r and ϵ_t in terms of radial displacement (u) and substituting values of ϵ_z and σ_r and σ_t from Eqs. (13), (16) and (18) respectively ($\sigma_z = 0$) yields the following differential equation:

$$\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r} = \frac{\sigma_y}{E}\left\{(1 - 2\mu)\left[2.16 \ln \frac{r}{\rho} + 1{\cdot}08 - \frac{2(b^2 - \rho^2)}{\sqrt{(3b^4 + \rho^4)}}\right] + \frac{2\mu\rho^2}{\sqrt{(3b^4 + \rho^4)}}\right\} \tag{20}$$